belief state, consisting of the first few states examined, is also unsolvable. In some cases, this leads to a speedup proportional to the size of the belief states, which may themselves be as large as the physical state space itself.

4.4.2 Searching in partially observable environments

Many problems cannot be solved without sensing. For example, the sensorless 8-puzzle is impossible. On the other hand, a little bit of sensing can go a long way: we can solve 8-puzzles if we can see just the upper-left corner square. The solution involves moving each tile in turn into the observable square and keeping track of its location from then on.

For a partially observable problem, the problem specification will specify a $\text{PERCEPT}(s)$ function that returns the percept received by the agent in a given state. If sensing is nondeterministic, then we can use a PERCEPTS function that returns a set of possible percepts. For fully observable problems, $\text{PERCEPT}(s) = s$ for every state s, and for sensorless problems $\text{PERCEPT}(s) = \mathit{null}$.

Consider a local-sensing vacuum world, in which the agent has a position sensor that yields the percept L in the left square, and R in the right square, and a dirt sensor that yields *Dirty* when the current square is dirty and *Clean* when it is clean. Thus, the PERCEPT in state 1 is $[L, \mathit{Dirty}]$. With partial observability, it will usually be the case that several states produce the same percept; state 3 will also produce $[L, \mathit{Dirty}]$. Hence, given this initial percept, the initial belief state will be $\{1, 3\}$. We can think of the transition model between belief states for partially observable problems as occurring in three stages, as shown in Figure 4.15:

- The **prediction** stage computes the belief state resulting from the action, $\text{RESULT}(b, a)$, exactly as we did with sensorless problems. To emphasize that this is a prediction, we use the notation $\hat{b} = \text{RESULT}(b, a)$, where the "hat" over the b means "estimated," and we also use $\text{PREDICT}(b, a)$ as a synonym for $\text{RESULT}(b, a)$.
- The **possible percepts** stage computes the set of percepts that could be observed in the predicted belief state (using the letter o for observation):

$$\text{POSSIBLE-PERCEPTS}(\hat{b}) = \{o : o = \text{PERCEPT}(s) \text{ and } s \in \hat{b}\}.$$

- The **update** stage computes, for each possible percept, the belief state that would result from the percept. The updated belief state b_o is the set of states in $\hat{b}$ that could have produced the percept:

$$b_o = \text{UPDATE}(\hat{b}, o) = \{s : o = \text{PERCEPT}(s) \text{ and } s \in \hat{b}\}.$$

  The agent needs to deal with *possible* percepts at planning time, because it won't know the *actual* percepts until it executes the plan. Notice that nondeterminism in the physical environment can enlarge the belief state in the prediction stage, but each updated belief state b_o can be no larger than the predicted belief state $\hat{b}$; observations can only help reduce uncertainty. Moreover, for deterministic sensing, the belief states for the different possible percepts will be disjoint, forming a *partition* of the original predicted belief state.

Putting these three stages together, we obtain the possible belief states resulting from a given action and the subsequent possible percepts:

$$\begin{aligned} \text{RESULTS}(b, a) = \{b_o : b_o &= \text{UPDATE}(\text{PREDICT}(b, a), o) \text{ and} \\ o &\in \text{POSSIBLE-PERCEPTS}(\text{PREDICT}(b, a))\}. \end{aligned} \quad (4.5)$$