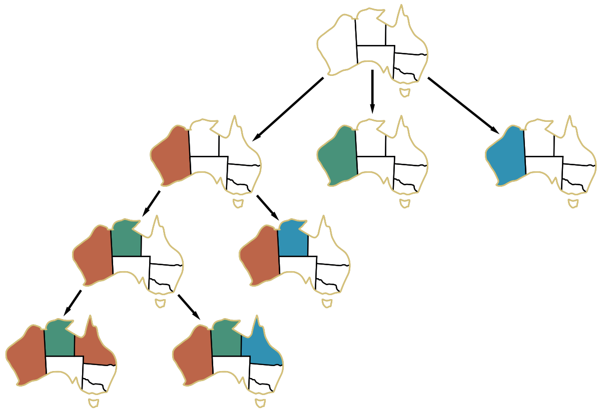

Figure 6.6 Part of the search tree for the map-coloring problem in Figure 6.1.

6.3.1 Variable and value ordering

The backtracking algorithm contains the line

$var \leftarrow$ SELECT-UNASSIGNED-VARIABLE(*csp*, *assignment*) .

The simplest strategy for SELECT-UNASSIGNED-VARIABLE is static ordering: choose the variables in order, $\{X_1, X_2, \ldots\}$. The next simplest is to choose randomly. Neither strategy is optimal. For example, after the assignments for $WA = red$ and $NT = green$ in Figure 6.6, there is only one possible value for SA, so it makes sense to assign $SA = blue$ next rather than assigning Q. In fact, after SA is assigned, the choices for Q, NSW, and V are all forced.

This intuitive idea—choosing the variable with the fewest "legal" values—is called the **minimum-remaining-values** (MRV) heuristic. It also has been called the "most constrained variable" or "fail-first" heuristic, the latter because it picks a variable that is most likely to cause a failure soon, thereby pruning the search tree. If some variable X has no legal values left, the MRV heuristic will select X and failure will be detected immediately—avoiding pointless searches through other variables. The MRV heuristic usually performs better than a random or static ordering, sometimes by orders of magnitude, although the results vary depending on the problem.

Minimum-remaining-values

The MRV heuristic doesn't help at all in choosing the first region to color in Australia, because initially every region has three legal colors. In this case, the **degree heuristic** comes in handy. It attempts to reduce the branching factor on future choices by selecting the variable that is involved in the largest number of constraints on other unassigned variables. In Figure 6.1, SA is the variable with highest degree, 5; the other variables have degree 2 or 3, except for T, which has degree 0. It we assign SA first, we can then go around the five mainland regions in clockwise or counterclockwise order and assign each one a color that is different than SA and different than the previous region. The minimum-remaining-values heuristic is usually a more powerful guide, but the degree heuristic can be useful as a tie-breaker.

Degree heuristic

Once a variable has been selected, the algorithm must decide on the order in which to examine its values. The **least-constraining-value** heuristic is effective for this. It prefers the value that rules out the fewest choices for the neighboring variables in the constraint

Least-constraining-value