successor-state axiom can say that the tub is empty before the action and full when the action is done, but it can't talk about what happens *during* the action. It also can't easily describe two actions happening at the same time—such as brushing one's teeth while waiting for the tub to fill. To handle such cases we introduce an approach known as **event calculus**.

Event calculus

The objects of event calculus are events, fluents, and time points. $At(Shankar, Berkeley)$ is a fluent: an object that refers to the fact of Shankar being in Berkeley. The event E_1 of Shankar flying from San Francisco to Washington, D.C., is described as

$$E_1 \in Flyings \land Flyer(E_1, Shankar) \land Origin(E_1, SF) \land Destination(E_1, DC) .$$

where *Flyings* is the category of all flying events. By reifying events we make it possible to add any amount of arbitrary information about them. For example, we can say that Shankar's flight was bumpy with $Bumpy(E_1)$. In an ontology where events are n-ary predicates, there would be no way to add extra information like this; moving to an $n+1$-ary predicate isn't a scalable solution.

To assert that a fluent is actually true starting at some point in time t_1 and continuing to time t_2, we use the predicate T, as in $T(At(Shankar, Berkeley), t_1, t_2)$. Similarly, we use $Happens(E_1, t_1, t_2)$ to say that the event E_1 actually happened, starting at time t_1 and ending at time t_2. The complete set of predicates for one version of the event calculus[4] is:

$T(f, t_1, t_2)$	Fluent f is true for all times between t_1 and t_2
$Happens(e, t_1, t_2)$	Event e starts at time t_1 and ends at t_2
$Initiates(e, f, t)$	Event e causes fluent f to become true at time t
$Terminates(e, f, t)$	Event e causes fluent f to cease to be true at time t
$Initiated(f, t_1, t_2)$	Fluent f become true at some point between t_1 and t_2
$Terminated(f, t_1, t_2)$	Fluent f cease to be true at some point between t_1 and t_2
$t_1 < t_2$	Time point t_1 occurs before time t_2

We can describe the effects of a flying event:

$$\begin{aligned}&E = Flyings(a, here, there) \land Happens(E, t_1, t_2) \Rightarrow \\ &\quad Terminates(E, At(a, here), t_1) \land Initiates(E, At(a, there), t_2)\end{aligned}$$

We assume a distinguished event, *Start*, that describes the initial state by saying which fluents are true (using *Initiates*) or false (using *Terminated*) at the start time. We can then describe what fluents are true at what points in time with a pair of axioms for T and $\neg T$ that follow the same general format as the successor-state axioms: Assume an event happens between time t_1 and t_3, and at t_2 somewhere in that time interval the event changes the value of fluent f, either initiating it (making it true) or terminating it (making it false). Then at time t_4 in the future, if no other intervening event has changed the fluent (either terminated or initiated it, respectively), then the fluent will have maintained its value. Formally, the axioms are:

$$\begin{aligned}&Happens(e, t_1, t_3) \land Initiates(e, f, t_2) \land \neg Terminated(f, t_2, t_4) \land t_1 \leq t_2 \leq t_3 \leq t_4 \Rightarrow \\ &\qquad T(f, t_2, t_4) \\ &Happens(e, t_1, t_3) \land Terminates(e, f, t_2) \land \neg Initiated(f, t_2, t_4)) \land t_1 \leq t_2 \leq t_3 \leq t_4 \Rightarrow \\ &\qquad \neg T(f, t_2, t_4)\end{aligned}$$

[4] Our version is based on Shanahan (1999), but with some alterations.