

# CHAPTER 13

## PROBABILISTIC REASONING

*In which we explain how to build efficient network models to reason under uncertainty according to the laws of probability theory, and how to distinguish between correlation and causality.*

Chapter 12 introduced the basic elements of probability theory and noted the importance of independence and conditional independence relationships in simplifying probabilistic representations of the world. This chapter introduces a systematic way to represent such relationships explicitly in the form of **Bayesian networks**. We define the syntax and semantics of these networks and show how they can be used to capture uncertain knowledge in a natural and efficient way. We then show how probabilistic inference, although computationally intractable in the worst case, can be done efficiently in many practical situations. We also describe a variety of approximate inference algorithms that are often applicable when exact inference is infeasible. Chapter 15 extends the basic ideas of Bayesian networks to more expressive formal languages for defining probability models.

### 13.1 Representing Knowledge in an Uncertain Domain

In Chapter 12, we saw that the full joint probability distribution can answer any question about the domain, but can become intractably large as the number of variables grows. Furthermore, specifying probabilities for possible worlds one by one is unnatural and tedious.

We also saw that independence and conditional independence relationships among variables can greatly reduce the number of probabilities that need to be specified in order to define the full joint distribution. This section introduces a data structure called a **Bayesian network**<sup>1</sup> to represent the dependencies among variables. Bayesian networks can represent essentially *any* full joint probability distribution and in many cases can do so very concisely.

A Bayesian network is a directed graph in which each node is annotated with quantitative probability information. The full specification is as follows:

1. Each node corresponds to a random variable, which may be discrete or continuous.
2. Directed links or arrows connect pairs of nodes. If there is an arrow from node  $X$  to node  $Y$ ,  $X$  is said to be a *parent* of  $Y$ . The graph has no directed cycles and hence is a directed acyclic graph, or DAG.
3. Each node  $X_i$  has associated probability information  $\theta(X_i | Parents(X_i))$  that quantifies the effect of the parents on the node using a finite number of **parameters**.

<sup>1</sup> Bayesian networks, often abbreviated to “Bayes net,” were called **belief networks** in the 1980s and 1990s. A **causal network** is a Bayes net with additional constraints on the meaning of the arrows (see Section 13.5). The term **graphical model** refers to a broader class that includes Bayesian networks.

Bayesian network

Parameter