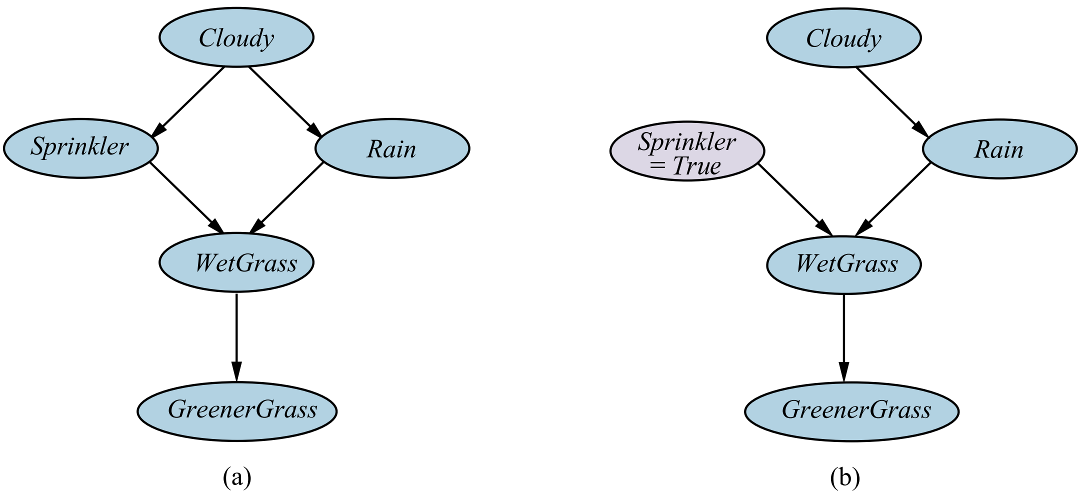

Figure 13.23 (a) A causal Bayesian network representing cause–effect relations among five variables. (b) The network after performing the action "turn *Sprinkler* on."

13.5.1 Representing actions: The *do*-operator

Consider again the *Sprinkler* story of Figure 13.23(a). According to the standard semantics of Bayes nets, the joint distribution of the five variables is given by a product of five conditional distributions:

$$P(c,r,s,w,g) = P(c)\, P(r\,|\,c)\, P(s\,|\,c)\, P(w\,|\,r,s)\, P(g\,|\,w) \tag{13.14}$$

where we have abbreviated each variable name by its first letter. As a system of structural equations, the model looks like this:

$$\begin{aligned} C &= f_C(U_C) \\ R &= f_R(C,U_R) \\ S &= f_S(C,U_S) \\ W &= f_W(R,S,U_W) \\ G &= f_G(W,U_G) \end{aligned} \tag{13.15}$$

where, without loss of generality, f_C can be the identity function. The U-variables in these equations represent **unmodeled variables**, also called **error terms** or **disturbances**, that perturb the functional relationship between each variable and its parents. For example, U_W may represent another potential source of wetness, in addition to *Sprinkler* and *Rain*—perhaps *MorningDew* or *FirefightingHelicopter*.

Unmodeled variable

If all the U-variables are mutually independent random variables with suitably chosen priors, the joint distribution in Equation (13.14) can be represented exactly by the structural equations in Equation (13.15). Thus, a system of stochastic relationships can be captured by a system of deterministic relationships, each of which is affected by an exogenous disturbance. However, the system of structural equations gives us more than that: it allows us to predict how *interventions* will affect the operation of the system and hence the observable consequences of those interventions. This is not possible given just the joint distribution.