The probability terms in the sum are obtained by computation on the original network, by any of the standard inference algorithms. This equation is known as an adjustment formula; it is a probability-weighted average of the influence of \( X_j \) and its parents on \( X_i \), where the weights are the priors on the parent values. The effects of intervening on multiple variables can be computed by imagining that the individual interventions happen in sequence, each one in turn deleting the causal influences on a variable and yielding a new, mutilated model.

### 13.5.2 The back-door criterion

The ability to predict the effect of any intervention is a remarkable result, but it does require accurate knowledge of the necessary conditional distributions in the model, particularly \( P(x_j \mid \text{parents}(X_j)) \). In many real-world settings, however, this is too much to ask. For example, we know that “genetic factors” play a role in obesity, but we do not know which genes play a role or the precise nature of their effects. Even in the simple story of Mary’s sprinkler decisions (Figure 13.15, which also applies in Figure 13.23(a)), we might know that she checks the weather before deciding whether to turn on the sprinkler, but we might not know how she makes her decision.

The specific reason this is problematic in this instance is that we would like to predict the effect of turning on the sprinkler on a downstream variable such as GreenerGrass, but the adjustment formula (Equation (13.20)) must take into account not only the direct route from Sprinkler, but also the “back door” route via Cloudy and Rain. If we knew the value of Rain, this back-door path would be blocked—which suggests that there might be a way to write an adjustment formula that conditions on Rain instead of Cloudy. And indeed this is possible:

\[
P(g \mid \text{do}(S=True)) = \sum_r P(g \mid S=True, r)P(r)
\]  

(13.21)

In general, if we wish to find the effect of \( \text{do}(X_j=x_{jk}) \) on a variable \( X_i \), the back-door criterion allows us to write an adjustment formula that conditions on any set of variables \( Z \) that closes the back door, so to speak. In more technical language, we want a set \( Z \) such that \( X_i \) is conditionally independent of \( \text{Parents}(X_j) \) given \( X_j \) and \( Z \). This is a straightforward application of d-separation (see page 419).

The back-door criterion is a basic building block for a theory of causal reasoning that has emerged in the past two decades. It provides a way to argue against a century of statistical dogma asserting that only a randomized controlled trial can provide causal information. The theory has provided conceptual tools and algorithms for causal analysis in a wide range of non-experimental and quasi-experimental settings; for computing probabilities on counterfactual statements (“if this had happened instead, what would the probability have been?”); for determining when findings in one population can be transferred to another; and for handling all forms of missing data when learning probability models.

### Summary

This chapter has described Bayesian networks, a well-developed representation for uncertain knowledge. Bayesian networks play a role roughly analogous to that of propositional logic for definite knowledge.