For example, suppose we *turn the sprinkler on*—that is, if we (who are, by definition, not part of the causal processes described by the model) *intervene* to impose the condition $Sprinkler = true$. In the notation of the **do-calculus**, which is a key part of the theory of causal networks, this is written as $do(Sprinkler = true)$. Once done, this means that the sprinkler variable is no longer dependent on whether it's a cloudy day. We therefore delete the equation $S = f_S(C, U_S)$ from the system of structural equations and replace it with $S = true$, giving us

Do-calculus

$$\begin{aligned} C &= f_C(U_C) \\ R &= f_R(C, U_R) \\ S &= true \\ W &= f_W(R, S, U_W) \\ G &= f_G(W, U_G). \end{aligned} \tag{13.16}$$

From these equations, we obtain the new joint distribution for the remaining variables conditioned on $do(Sprinkler = true)$:

$$P(c, r, w, g \mid do(S = true)) = P(c)\, P(r \mid c)\, P(w \mid r, s = true)\, P(g \mid w) \tag{13.17}$$

This corresponds to the "mutilated" network in Figure 13.23(b). From Equation (13.17), we see that the only variables whose probabilities change are *WetGrass* and *GreenerGrass*, that is, the descendants of the manipulated variable *Sprinkler*.

Note the difference between conditioning on the *action* $do(Sprinkler = true)$ in the original network and conditioning on the *observation* $Sprinkler = true$. The original network tells us that the sprinkler is less likely to be on when the weather is cloudy, so if we *observe* the sprinkler to be on, that reduces the probability that the weather is cloudy. But common sense tells us that if we (operating from outside the world, so to speak) reach in and turn on the sprinkler, that doesn't affect the weather or provide new information about what the weather is like that day. As shown in Figure 13.23(b), intervening breaks the normal causal link between the weather and the sprinkler. This prevents any influence flowing backward from *Sprinkler* to *Cloudy*. Thus, conditioning on $do(Sprinkler = true)$ in the original graph is equivalent to conditioning on $Sprinkler = true$ in the mutilated graph.

A similar approach can be taken to analyze the effect of $do(X_j = x_{jk})$ in a general causal network with variables $X_1, \ldots, X_n$. The network corresponds to a joint distribution defined in the usual way (see Equation (13.2)):

$$P(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid parents(X_i)). \tag{13.18}$$

After applying $do(X_j = x_{jk})$, the new joint distribution $P_{x_{jk}}$ simply omits the factor for X_j:

$$P_{x_{jk}}(x_1, \ldots, x_n) = \begin{cases} \prod_{i \neq j} P(x_i \mid parents(X_i)) = \frac{P(x_1, \ldots, x_n)}{P(x_j \mid parents(X_j))} & \text{if } x_j = x_{jk} \\ 0 & \text{if } x_j \neq x_{jk} \end{cases} \tag{13.19}$$

This follows from the fact that setting X_j to a particular value x_{jk} corresponds to deleting the equation $X_j = f_j(Parents(X_j), U_j)$ from the system of structural equations and replacing it with $X_j = x_{jk}$. With a bit more algebraic manipulation, one can derive a formula for the effect of setting variable X_j on any other variable X_i:

$$\begin{aligned} P(X_i = x_i \mid do(X_j = x_{jk})) &= P_{x_{jk}}(X_i = x_i) \\ &= \sum_{parents(X_j)} P(x_i \mid x_{jk}, parents(X_j)) P(parents(X_j)). \end{aligned} \tag{13.20}$$